

THERMAL STABILITY OF A DISTURBED FLUID FLOW

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Stability of periodic solutions of a non-self-similar nonlinear problem is studied. The problem describes the thermal state of an axial fluid flow with continuously distributed sources of heat. The flow experiences the action of external low-amplitude perturbations changing in time in accordance with known periodic laws. The spectral problem is solved by the method of parametrix, and the critical conditions of the thermal explosion are determined in the linear approximation. Stability of the periodic solution at the critical point is evaluated using the known theorem of factorization, which takes into account the effect of nonlinear terms of the heat-balance equation. The calculation results show that the periodic solution is stable if the total action of external periodic perturbations at the critical point is directed to reduction of the fluid-flow temperature.

1. Formulation of the Problem. In the thermal explosion theory [1, 2], the physicochemical state of a continuous medium with continuously distributed sources of heat is described by a system of equations of diffusion, heat conduction, and kinetics. The case where the temperature field of the medium and the concentration field of one of the reacting components limiting the exothermic reaction rate are similar is most comprehensively studied. In this case, the diffusion equation is identical to the heat-conduction equation, and the problem of the thermal state of the medium reduces to one equation in partial derivatives that satisfies some boundary conditions. In dimensionless coordinates, the problem for a one-dimensional fluid flow with continuously distributed sources of heat has the form

$$\frac{\partial \theta}{\partial t} = a \frac{\partial^2 \theta}{\partial x^2} - u \frac{\partial \theta}{\partial x} + \varphi(\theta) \quad (t_0 \leq t \leq \infty, 0 \leq x \leq l, u \geq 0); \quad (1.1)$$

$$\theta(0, t) = 0, \quad \frac{\partial \theta(l, t)}{\partial x} = 0, \quad (1.2)$$

where x is the coordinate, t is the time, θ is the temperature, a is the thermal diffusivity, u is the flow velocity directed toward positive values of the x coordinate, t_0 is the initial time, l is the coordinate of the right boundary of the domain, and $\varphi(\theta)$ is the source function characterizing the heat-release intensity in the flow [generally, according to the Arrhenius law, $\varphi(\theta) = z \exp(\theta(1 + \beta\theta)^{-1})$, where $\beta = \text{const} \geq 0$ and z is the preexponent]. In problem (1), (2), the transition to dimensional parameters and coordinates is performed by means of commonly accepted coordinate and time scales given in [1, 2].

In [3], stability of the solution of the nonlinear problem (1.1), (1.2) was analyzed for time-independent thermal diffusivity a , flow velocity u , and coefficients b_k in the expansion of the source function into a series

$$\varphi(\theta) = \sum_{k=0}^{\infty} b_k \theta^k, \quad b_k = \frac{1}{k!} \frac{\partial^k \varphi(0)}{\partial \theta^k}. \quad (1.3)$$

We consider problem (1.1), (1.2) for the case where the fluid flow is affected by time-dependent perturbations considered as external control parameters. Let

$$a = a_0 + a_1(t), \quad u = u_0 + u_1(t), \quad b_0 = b_0(t), \quad b_k = b_{k0} + b_{k1}(t) \quad (k = 1, 2, \dots), \quad (1.4)$$

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where a_0 , u_0 , and b_{k_0} are constants, $a_1(t)$, $u_1(t)$, $b_0(t)$, and $b_{k_1}(t)$ are continuous periodic functions of time that satisfy the initial conditions $a_1(t_0) = 0$, $u_1(t_0) = 0$, $b_0(t_0) = 0$, and $b_{k_1}(t_0) = 0$.

The problem is to determine stability of the solution of the unsteady problem (1.1), (1.2) with allowance for expansion (1.3) and coefficients (1.4). The free coefficient of series (1.3) is written in the form $b_0 = \Delta b_0(t)$, where Δ is a function that takes the values 0 or 1. First, we study stability of bifurcation solutions for $\Delta = 0$ and, then, stability of isolated solutions for $\Delta = 1$.

2. Stability of the Solution of the Linearized Problem. We write the linearized equation (1.1) for $\Delta = 0$ in an operator form:

$$L\theta = 0, \quad L = (a_0 + a_1(t))\frac{\partial^2}{\partial x^2} - (u_0 + u_1(t))\frac{\partial}{\partial x} + b_{10} + b_{11}(t) - \frac{\partial}{\partial t}. \quad (2.1)$$

To construct the fundamental solution of Eq. (2.1), we use the method of parametrix [4]. As the parametrix $G(x, t; \xi, \tau)$, we use the following fundamental solution of the equation

$$L_0\theta = 0, \quad L_0 = a_0\frac{\partial^2}{\partial x^2} - u_0\frac{\partial}{\partial x} + b_{10} - \frac{\partial}{\partial t}$$

satisfying the boundary conditions (1.2):

$$G(x, t; \xi, \tau) = \sum_{i=1}^{\infty} \exp\left(\sigma_i(t - \tau) + \frac{u_0(x - \xi)}{2}\right) \sin \lambda_i x \sin \lambda_i \xi.$$

Here τ is the time within the interval (t_0, t) , ξ is the coordinate within the interval $(0, l)$, λ_n are the positive roots of the transcendental equation $\tan \lambda l = -2\lambda/u_0$ located in ascending order ($\lambda_1 < \lambda_2 < \dots$) and $\sigma_n = b_{10} - u_0^2/4 - \lambda_n^2$ is the eigenvalue of the operator L_0 . By definition, $G(x, t; \xi, \tau)$ as a function of x, t for fixed ξ, τ satisfies the equation $L_0\theta = 0$.

Let the flow temperature at the initial time $t_0 = 0$ be distributed according to the law

$$\theta(x, 0) = \theta_0(x) = \exp\left(\frac{u_0 x}{2}\right) \sum_{n=1}^{\infty} \sin \lambda_n x. \quad (2.2)$$

Then, the function

$$\Theta^0(x, t) = \int_0^l G(x, t; \xi, 0) \theta_0(\xi) d\xi$$

is the solution of the Cauchy problem [equation $L_0\theta = 0$ with conditions (1.2) and (2.2)]. After integration, we obtain

$$\Theta^0(x, t) = \sum_{n=1}^{\infty} \exp\left(\sigma_n t + \frac{u_0 x}{2}\right) \sin \lambda_n x. \quad (2.3)$$

Since the spectrum of the operator L_0 satisfies the inequalities $\sigma_1 > \sigma_n$ for $n > 1$, it follows from (2.3) that $\Theta^0(x, t) \rightarrow 0$ for $\sigma_1 < 0$, and the steady solution of the equation $L_0\theta = 0$ is stable; for $\sigma_1 > 0$, we have $\Theta^0(x, t) \rightarrow \infty$, and the solution is unstable. Stability of the steady solution at the critical point $\sigma_1 = 0$ for an algebraically simple σ_1 was investigated in [3].

The right side of (2.3) may be considered as the sum of projections of the solution onto the space of eigenfunctions $y_n^0 = \exp(u_0 x/2) \sin \lambda_n x$ ($n = 1, 2, \dots$) of the operator L_0 . Stability of the solution of the equation $L\theta = 0$ at the point $\sigma_1 = 0$ is determined by the projection onto the eigenfunction y_1^0 , since the remaining projections decrease exponentially with time. Therefore, in constructing the solution of Eq. (2.1) at the point $\sigma_1 = 0$, we retain only its projection onto y_1^0 .

The fundamental solution $\Gamma(x, t; \xi, \tau)$ of Eq. (2.1) has the form [4]

$$\Gamma(x, t; \xi, \tau) = G(x, t; \xi, \tau) + \int_{\tau}^t \int_0^l G(x, t; \eta, \zeta) \Phi(\eta, \zeta; \xi, \tau) d\eta d\zeta,$$

where the function $\Phi(x, t; \xi, \tau)$ is the solution of the integral Volterra equation

$$\Phi(x, t; \xi, \tau) = LG(x, t; \xi, \tau) + \int_{\tau}^t \int_0^l LG(x, t; \eta, \zeta) \Phi(\eta, \zeta; \xi, \tau) d\eta d\zeta \quad (2.4)$$

with the kernel

$$LG(x, t; \eta, \zeta) = \left(a_1(t) \frac{\partial^2}{\partial x^2} - u_1(t) \frac{\partial}{\partial x} + b_{11}(t) \right) G(x, t; \eta, \zeta).$$

The solution of Eq. (2.4) can be represented as the sum

$$\Phi(x, t; \xi, \tau) = \sum_{k=1}^{\infty} (LG)_k(x, t; \xi, \tau), \quad (2.5)$$

where $(LG)_{k+1}(x, t; \xi, \tau) = \int_{\tau}^t \int_0^l [LG(x, t; \eta, \zeta)] (LG)_k(\eta, \zeta; \xi, \tau) d\eta d\zeta$ and $(LG)_1(x, t; \xi, \tau) = LG(x, t; \xi, \tau)$.

Convergence of series (2.5) for continuous (according to Hölder) functions $a_1(t)$, $u_1(t)$, and $b_{11}(t)$ was proved in [4]. If we identify the Hölder continuity coefficient A with the maximum amplitude of periodic functions $a_1(t)$, $u_1(t)$, and $b_{11}(t)$, then we obtain $(LG)_k(x, t; \xi, \tau) = O(A^k)$. It follows from here that series (2.5) rapidly converges for low amplitudes of perturbations of fluid-flow parameters.

The solution of the disturbed Cauchy problem [equations (2.1) with conditions (1.2) and (2.2)] is the function

$$\Theta(x, t) = \int_0^l \Gamma(x, t; \xi, 0) \theta_0(\xi) d\xi,$$

which is the sum

$$\Theta(x, t) = \Theta^0(x, t) + \Theta^1(x, t), \quad (2.6)$$

where $\Theta^1(x, t)$ is the part of the solution caused by the perturbations $a_1(t)$, $u_1(t)$, and $b_{k1}(t)$.

In the case of low amplitudes of perturbations periodic in time, we can ignore the terms of series (2.5) that are nonlinear with respect to the amplitudes and represent the function $\Theta^1(x, t)$ from (2.6) in the form

$$\Theta^1(x, t) = \exp\left(\sigma_1 t + \frac{u_0 x}{2}\right) \sin(\lambda_1 x) \psi(l, t) + O(A^2), \quad (2.7)$$

where

$$\psi(l, t) = \frac{lu_0^2 + 2u_0 - 4\lambda_1^2 l}{8} \int_0^t a_1(\zeta) d\zeta - \frac{lu_0 + 2}{4} \int_0^t u_1(\zeta) d\zeta + \frac{\lambda_1 l - \sin \lambda_1 l \cos \lambda_1 l}{2\lambda_1} \int_0^t b_{11}(\zeta) d\zeta.$$

As is mentioned above, Eq. (2.7) contains only the projection of the solution on the eigenfunction

$$y_1(x, t) = \exp(u_0 x / 2) \sin(\lambda_1 x) \psi(l, t), \quad (2.8)$$

corresponding to the maximum eigenvalue of σ_1 . If the antiderivatives of the functions $a_1(t)$, $u_1(t)$, and $b_{11}(t)$ in (2.7) are periodic, then $\psi(l, t) = \psi(l, t + T)$, and the solution (2.6) at the point $\sigma_1 = 0$ is also periodic: $\Theta(x, t) = \Theta(x, t + T)$. Generally speaking, it is not important how the function $\psi(l, t)$ is obtained. It is only necessary that it takes identical values in identical time periods T counted from the initial time.

Stability of the solution (2.6) depends on the sign of the eigenvalue of the parameter σ_1 , which is identified with the parameter $\mu \in (-\infty, \infty)$. The solution (2.6) definitely loses its stability when μ passes through zero on its way from the negative region to the positive one. The critical point $\mu = 0$ corresponds to the critical parameters of the fluid flow; when the change in the parameters involves an increase in μ , the thermal explosion occurs.

3. Stability of the Periodic Solution. To determine stability of the periodic solution of the problem at the critical point $\mu = 0$, we have to evaluate the influence of nonlinear terms in the right side of Eq. (1.1) on the parameter μ . We assume that $\Delta = 0$ and $\varphi(\theta) = b_1 \theta + b_2 \theta^2 + O(\theta^3)$ and write Eq. (1.1) in an operator form

$$F(t, \theta, \mu) \equiv L(\mu)\theta + (b_{20} + b_{21}(t))\theta^2 + O(\theta^3) = 0, \quad (3.1)$$

where the linear operator

$$L(\mu) = (a_0 + a_1(t)) \frac{\partial^2}{\partial x^2} - (u_0 + u_1(t)) \frac{\partial}{\partial x} + \mu + \frac{u_0^2}{4} + \lambda_1^2 + b_{11}(t) - \frac{\partial}{\partial t}$$

is obtained by substitution of $b_{10} = \mu + u_0^2/4 + \lambda_1^2$ into (2.1) for the operator L . Periodic solutions $\theta(x, t) = \theta(x, t+T)$ branch off from the solution $\theta(x, t) = 0$ when μ , increasing, passes through zero. In accordance with [5], the solutions of the nonlinear equation (3.1) in the vicinity of the point $\theta = 0$, $\mu = 0$ can be constructed in the form of series in powers of the amplitude ε . The amplitude is the projection on the proper subspace associated with the conjugate eigenvector $y_1^*(x, t)$ that belongs to the eigenvalue $\sigma_1 = 0$ of the conjugate operator

$$L^*(\mu) = (a_0 + a_1(t)) \frac{\partial^2}{\partial x^2} + (u_0 + u_1(t)) \frac{\partial}{\partial x} + \mu + \frac{u_0^2}{4} + \lambda_1^2 + b_{11}(t) + \frac{\partial}{\partial t}.$$

Following the procedure used in determining the eigenfunctions of the operator $L(\mu)$, we can find the eigenfunctions y_k^* of the operator $L^*(\mu)$. In studying stability of periodic solutions, it is of interest to consider the conjugate eigenfunction corresponding to the maximum eigenvalue of σ_1 at the point $\mu = 0$:

$$y_1^* = B \exp(-u_0 x/2) \sin(\lambda_1 x) \psi^*(l, t). \quad (3.2)$$

Here

$$\psi^*(l, t) = \frac{lu_0^2 - 2u_0 - 4\lambda_1^2 l}{8} \int_0^t a_1(\zeta) d\zeta - \frac{lu_0 - 2}{4} \int_0^t u_1(\zeta) d\zeta + \frac{\lambda_1 l - \sin \lambda_1 l \cos \lambda_1 l}{2\lambda_1} \int_0^t b_{11}(\zeta) d\zeta$$

and B is the normalization factor. The normalization condition $\langle y_1, y_1^* \rangle = 1$ is fulfilled if

$$B = \frac{2\lambda_1 T}{\lambda_1 l - \sin \lambda_1 l \cos \lambda_1 l} \left[\int_0^T \int_0^l \psi(l, t) \psi^*(l, t) dt \right]^{-1}.$$

In the space of real eigenfunctions of the operator $L(\mu)$, for each pair of the functions y_n, y_m^* , we determine the scalar product

$$\langle y_n, y_m^* \rangle = \frac{1}{T} \int_0^T \int_0^l y_n y_m^* dx dt.$$

For arbitrary n and m , the orthogonality conditions are valid: $\langle y_n, y_m^* \rangle = \delta_{nm}$, where δ_{nm} is the Kronecker delta.

We use the scalar product to determine the amplitude ε as the projection

$$\varepsilon = \langle \theta, y_1^* \rangle = \frac{1}{T} \int_0^T \int_0^l y_1 y_1^* dx dt.$$

We seek the solution of Eq. (3.1) in the form of series in powers of the amplitude

$$\theta(\varepsilon) = \sum_{k=1}^{\infty} \frac{\varepsilon^k \theta_k}{k!}, \quad \mu(\varepsilon) = \sum_{k=1}^{\infty} \frac{\varepsilon^k \mu_k}{k!}. \quad (3.3)$$

The unknown functions θ_k and the coefficients μ_k satisfy the equations obtained by substitution of series (3.3) into Eq. (3.1) and equating to zero the terms at the independent powers of the amplitude ε . Stability of the solution of Eq. (3.1), which satisfies the boundary conditions (2.1) and the initial condition $\theta(x, 0) = 0$, is determined at the critical point $\mu = 0$ by the first nonzero coefficient μ_k from the series μ_1, μ_2, \dots . To determine the first coefficient μ_1 , one has to solve equations for the first and second powers of the amplitude:

$$L(0)\theta_1 = 0, \quad (3.4)$$

$$L(0)\theta_2 + 2\mu_1\theta_1 + 2(b_{20} + b_{21}(t))\theta_1^2 = 0. \quad (3.5)$$

Equation (3.4) has the only solution $\theta_1 = y_1(x, t)$, which coincides with (2.7) for $\mu = \sigma_1 = 0$. Equation (3.5) can be solved when and only when its inhomogeneous terms are orthogonal to the proper subspace of the operator $L(0)$ associated with the conjugate eigenvector y_1^* (Fredholm's alternative). Hence, we have

$$\mu_1 \langle \theta_1, y_1^* \rangle + \langle (b_{20} + b_{21}(t))\theta_1^2, y_1^* \rangle = 0,$$

and the coefficient μ_1 is determined with allowance for the solution of Eq. (3.4) and normalization $\langle y_1, y_1^* \rangle = 1$ in the form

$$\mu_1 = -\langle (b_{20} + b_{21}(t))y_1^2, y_1^* \rangle. \quad (3.6)$$

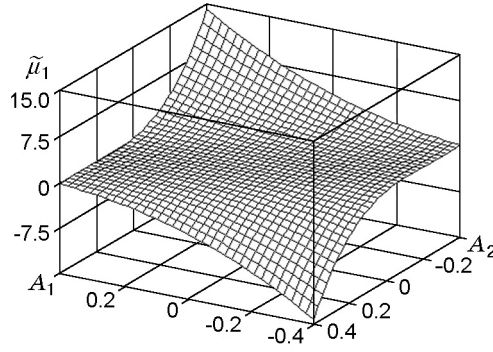


Fig. 1

In the general case, the co-factors of the scalar product in Eq. (3.6) determined in the rectangle $(0, l) \times (0, T)$ are not equal to zero and are not orthogonal to each other; hence, $\mu_1 \neq 0$. The latter inequality is valid for solutions of non-self-similar problems periodic in time only if $\partial^2 F(t, 0, 0)/\partial \theta^2 \neq 0$ and σ_1 is an algebraically simple isolated eigenvalue [5].

In accordance with the theorem of factorization [5], the solution of Eq. (3.1) with respect to a small perturbation $\tilde{\theta}(\varepsilon)$ in a small vicinity of the point $(\mu, \theta) = (0, \Theta(x, t))$ has the form

$$\tilde{\theta}(\varepsilon) = \exp(-\mu_1 \varepsilon t) y_1 \varepsilon. \quad (3.7)$$

It follows from (3.7) that the supercritical solution of the problem $\mu_1 > 0$ is stable, and the subcritical solution $\mu_1 < 0$ is unstable. The uniqueness of the solution of Eq. (3.7) is reached by normalization $\varepsilon = 1$.

4. Calculation Results. The time and coordinate scales in the classical problem of thermal explosion [1, 2] were chosen such that $a_{10} = 1$, $b_{10} = 1$, and $b_{20} = 0.5 - \beta$ for a zero-order exothermic reaction in the fluid. Let the free-stream velocity be $u_0 = 1$ and the coefficient be $\beta = 0.05$. For these values of the free-stream parameters, we obtain the critical values $\lambda_1 = 0.867$, $l = 2.418$, $\sigma_1 = 0$, and the eigenfunction $y_1^0 = \exp(x/2) \sin 0.867x$.

It should be noted that the system of two equations with respect to the periodic functions $\psi(l, t)$ and $\psi^*(l, t)$ contains three independent variables $a_1(t)$, $u_1(t)$, and $b_1(t)$; hence, it has an infinite set of solutions. Therefore, without loss of generality, we assume that

$$a_1(t) = 0, \quad u_1(t) = A_1 \sin t, \quad b_{n1}(t) = b_{n0} A_2 \sin t \quad (n = 1, 2, \dots). \quad (4.1)$$

The perturbation amplitudes A_1 and A_2 are not defined in more detail yet. Substituting relations (4.1) into Eqs. (2.8) and (3.2), we find the eigenfunctions of the operators $L(\mu)$ and $L^*(\mu)$:

$$y_1 = (1.105A_1(\cos t - 1) + 1.459A_2(1 - \cos t)) \exp(x/2) \sin 0.867x. \quad (4.2)$$

$$y_1^* = B(0.105A_1(\cos t - 1) + 1.459A_2(1 - \cos t)) \exp(-x/2) \sin 0.867x.$$

Here $B = 4.303/(1.088A_1^2 + 20.071A_2^2 - 16.628A_1A_2)$ is a normalization factor.

Substitution of (4.1) and (4.2) into (3.6) yields the dependence of μ_1 on perturbation amplitudes A_1 and A_2 . Figure 1 shows the dependence $\tilde{\mu} = \mu(A_1, A_2)B^{-1}$ (substitution of $\tilde{\mu}_1$ for μ_1 in calculations eliminates division by zero at the point $A_1 = A_2 = 0$). The normalization factor B is always positive; therefore, the solution of Eq. (3.1) is stable for $\tilde{\mu}_1 > 0$ and unstable for $\tilde{\mu}_1 < 0$.

The calculation results show that $\mu_1 > 0$ and the solution is stable if the inequalities $du/dt > 0$ and $d\varphi/dt < 0$ are valid at the critical point $\mu = 0$. Physically, this means that, in the critical thermal state of the fluid, the temperature decreases due to the higher intensity of convective heat transfer through the right boundary and the lower intensity of the exothermic processes in the fluid. For $du/dt < 0$ and $d\varphi/dt > 0$, the solution is unstable, since convective heat transfer through the right boundary in the fluid decreases at the critical point, and the heat-release intensity in the fluid increases, which leads to an exponential increase in temperature. In the remaining cases, stability of the solution of Eq. (3.1) is determined by the relationship between A_1 and A_2 . Thus, for $A_1 = A_2 = 0.01$, we have $\mu_1 = -0.0349$, and the solution is unstable (Fig. 2a); for $A_1 = A_2 = -0.1$, we have

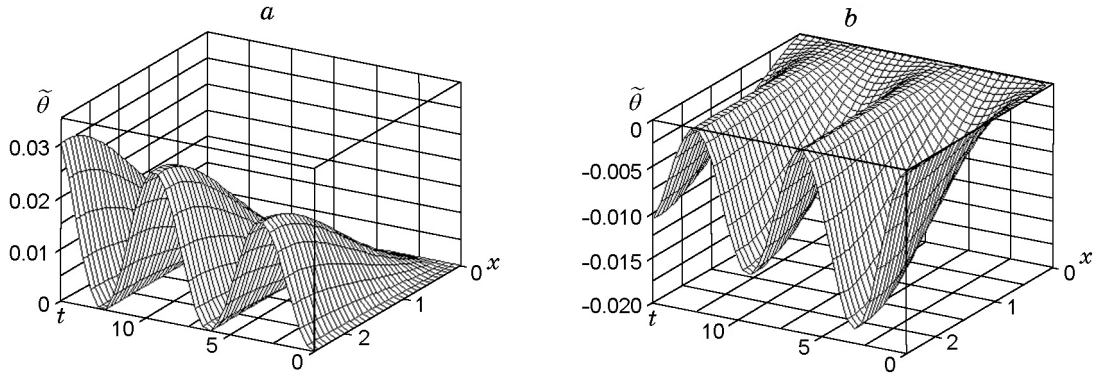


Fig. 2

$\mu_1 = 0.0349$, and the solution is stable (Fig. 2b). It should be noted that the reference point for the evolution of small perturbations of temperature $\tilde{\theta}$ in Fig. 2 is the solution of the corresponding steady problem $\Theta(x, t)$ at the critical point $\mu = \sigma_1 = 0$.

5. Effect of the Free Coefficient b_0 on Stability of the Solution. We return to Eq. (1.1) and rewrite it with allowance for expansion (1.3) in the form

$$F(t, \theta, \mu, \Delta) \equiv L(\mu)\theta + b_0\Delta(\mu, \varepsilon) + \sum_{n=2}^{\infty} (b_{n0} + b_{n1}(t))\theta^n = 0. \quad (5.1)$$

As was shown above, periodic solutions of Eq. (5.1) are divided at the double point into stable and unstable branches for $\Delta = 0$, and into isolated solutions that destroy bifurcation at the double point for $\Delta = 1$. It is shown [5] that isolated periodic solutions caused by the periodic defect b_0 may be found in the same manner as steady solutions if

$$\left\langle \frac{\partial F(t, 0, 0, 0)}{\partial \Delta}, y_1^* \right\rangle \neq 0. \quad (5.2)$$

Condition (5.2) and the theorem of the implicit function ensure the existence of solutions of the equations $F(t, \theta, \mu, \Delta) = 0$ and $\varepsilon = \langle \theta(\mu, \varepsilon), y_1^* \rangle = 1$ with respect to $\theta(\mu, \varepsilon)$ and $\Delta(\mu, \varepsilon)$.

Double differentiation of the function $F(t, \theta, \mu, \Delta)$ with respect to μ and ε at the point $(\mu, \varepsilon) = (0, 0)$ and subsequent use of Fredholm's alternative allow us to obtain, as in the steady case [3], two first nonzero terms in the expansion of $\Delta(\mu, \varepsilon)$ in powers of μ and ε :

$$\begin{aligned} \Delta(\mu, \varepsilon) &= \frac{1}{2} \left(\frac{\partial^2 \Delta(0, 0)}{\partial \varepsilon^2} \varepsilon^2 + 2 \frac{\partial^2 \Delta(0, 0)}{\partial \varepsilon \partial \mu} \varepsilon \mu \right), \\ \frac{\partial^2 \Delta(0, 0)}{\partial \varepsilon^2} &= - \left\langle \frac{\partial^2 F(t, 0, 0, 0)}{\partial \theta^2} y_1^2, y_1^* \right\rangle \left\langle \frac{\partial F(t, 0, 0, 0)}{\partial \Delta}, y_1^* \right\rangle^{-1}, \\ \frac{\partial^2 \Delta(0, 0)}{\partial \varepsilon \partial \mu} &= - \left\langle \frac{\partial^2 F(t, 0, 0, 0)}{\partial \theta \partial \mu} y_1, y_1^* \right\rangle \left\langle \frac{\partial F(t, 0, 0, 0)}{\partial \Delta}, y_1^* \right\rangle^{-1}. \end{aligned} \quad (5.3)$$

Taking into account that

$$\Delta(\mu, \varepsilon) = 1, \quad \frac{\partial F(t, 0, 0, 0)}{\partial \Delta} = b_0, \quad \frac{\partial^2 F(t, 0, 0)}{\partial \theta^2} = 2(b_{20} + b_{21}(t)), \quad \frac{\partial^2 F(t, 0, 0)}{\partial \theta \partial \mu} = 1,$$

from (5.3) we find $\mu = \mu_1 \varepsilon - \mu_{10} / \varepsilon$, where $\mu_{10} = \frac{1}{T} \int_0^T \int_0^l b_0 y_1^* dx dt$.

Stability of isolated solutions of Eq. (5.1) containing the periodic defect $b_0 \neq 0$ is determined as follows. If the derivative $\partial \mu / \partial \varepsilon$ at the point $(\varepsilon, \Delta) = (1, 0)$ retains its sign when passing to the point $(\varepsilon, \Delta) = (1, 1)$, then the solution stable at $\Delta = 0$ remains also stable for $\Delta = 1$. As the sign of the derivatives is changed, stable branches become unstable and vice versa.

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